A PROPOSED EUCLIDEAN ALTERNATIVE TO MINKOWSKI SPACETIME DIAGRAM.

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Abstract

The imaginary number present in the Minkowski spacetime diagram due to the non-euclidean (+++−) metric Lorentz transformation equations as derived from special relativity is briefly discussed. Based on a euclidean re-formulation of special relativity, a euclidean (++++) metric model is recovered by slightly modifying the 2nd postulate from the invariant light speed c to an invariant spacetime speed c of inertial frames. With a 4th dimensional proper time, the relativistic variations in this proposed model are governed by the functions of a circle expressed in trigonometric form in terms of a spacetime angle ϕ. Its possible usage as a convenient alternative to Minkowski diagram to investigate the Lorentz transformation is discussed.

Keywords: Special relativity, Euclidean 4-space-time, Lorentz transformation.

1.0 Minkowski spacetime diagram.

The Minkowski space-time (MST) diagram is a geometrical formulation of special relativity (SR) postulates that natural laws are invariant in any inertial reference frame and that light speed is an invariant (c) independent of the motion status of the source. The world is modeled as a 4-dimensional spacetime continuum and the diagram transforms the reference (x,y,z,t) coordinates to the (x',y',z',t') coordinates of the moving frame. Preserving the invariance of light speed implies $x^2 + y^2 + z^2 - c^2t^2 = 0$ and $x'^2 + y'^2 + z'^2 - c^2t'^2 = 0$ resulting in the non-euclidean (+++−) metric. For a body moving along the x-axis, $y = y' = 0$ and $z = z' = 0$ needing only two axes to represent the Lorentz transformation.

In the standard diagram, the two orthogonal axes of the reference coordinate system are the horizontal space-axis and a 4th dimensional vertical time-axis. The time-axis has the imaginary mathematical number i ($=\sqrt{-1}$) present in it as a consequence of the non-Euclidean (+++−) metric. For both axes to measure length displacement, time, t, is conveniently multiplied by c. With that, the displacement position in space and time of the reference and transformed coordinates are (x, ict) and (x', ict') respectively. The transformed axes skews as velocity v varies causing Minkowski squares to appear as rhombus and merges into the light worldline at $v = c$. The transformed axes coordinate units are calibrated by its intersections with the hyperbolic function curves corresponding to the non-euclidean metric transformation equation. A crucial aspect is the spacetime interval between any two events remain invariant under coordinate transformation.

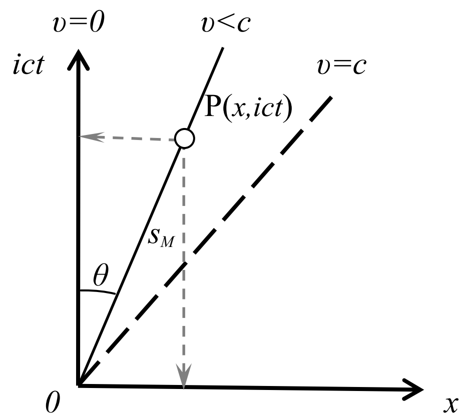

Figure 1: Trajectory of particle P in spacetime

The MST diagram (*Fig 1*) shows particle P's trajectory in spacetime (its worldline) with origin O tilted at a spacetime angle θ. At $\upsilon=0$, $\theta=0$ and the worldline is along the vertical *ict*-axis but tilts that uniquely corresponds to υ to a limiting velocity of c at $\theta = \frac{\pi}{4}$ radians, the light worldline. The worldline length, s_M (= OP), is P's displacement in spacetime . The subscript M in s_M denotes this displacement is in Minkowski spacetime. Its two components are the displacement in space along the x-axis and the displacement in time along the *ict*-axis. Although *Fig 1* only shows for positive x and t, the worldlines can be drawn to include negative values for displacement against the x-axis direction and negative values of time, t, by extending into its past trajectory.

2.0 Reviewing the MST diagram.

The introduction of SR elegantly explained the relativistic effects compared to the Lorentz explanation and also resolved the puzzling negative results of the Michelson-Morley experiment. The consistency of the MST diagram with SR provided the impetus for it to emerge as the dominant geometrical tool to investigate the Lorentz transformation. However, due the presence of i in the 4^{th} dimensional time-axis, the displacement in spacetime, s_M , is in a complex plane and the 4-rotation θ is not real. Although the mathematical significance of i is precise, a problem usually arises interpreting it in relation to its physical significance. An approach often adopted to go around this problem is to assume time as imaginary which permits the presence of i in the vertical time *ict*-axis to be ignored. On this assumption, the displacement in time is along a vertical *ct*-axis with the rotation θ interpreted as real where $\tan\theta = \frac{x}{ct} = \frac{\upsilon}{c}$.

Purely on the grounds of investigative curiosity, we next adopt an approach where time is assumed as real. On this assumption, the displacement in time is along a vertical *ict*-axis. Adopting this approach requires *i* to be interpreted as a constant similar in status to *c* in the MST diagram. Proceeding on this, from pythagorean theorem, $s_M{}^2 = x^2 + (ict)^2$ and in differential form, $ds_M{}^2 = dx^2 - c^2dt^2$. Transforming P's displacement in spacetime, s_M, and its two orthogonal component displacements in space, x, and time, ict, in terms of rate of displacement (i.e. speed) in spacetime, space and time respectively, the MST diagram (*Fig 1*) is re-presented as a velocity vector diagram (*Fig 2*).

The velocity in spacetime vector $\boldsymbol{\upsilon}_{\mathbf{st}}$ represents P's path in spacetime with its speed equal to the magnitude, $|\boldsymbol{\upsilon}_{\mathbf{st}}| = \upsilon_{\mathrm{st}}$. Its two vector components are the horizontal velocity in space vector $\boldsymbol{\upsilon}_{\mathbf{s}}$ (equivalent to coordinate velocity $\boldsymbol{\upsilon}$) and the vertical velocity in time vector $\boldsymbol{\upsilon}_{\mathbf{t}}$ with their speeds equal to their magnitudes $|\boldsymbol{\upsilon}_{\mathbf{s}}| = \upsilon_{\mathrm{s}} = \upsilon$ and $|\boldsymbol{\upsilon}_{\mathbf{t}}| = \upsilon_{\mathrm{t}}$. (The velocity vectors $\boldsymbol{\upsilon}_{\mathbf{st}}$, $\boldsymbol{\upsilon}_{\mathbf{s}}$ and $\boldsymbol{\upsilon}_{\mathbf{t}}$ are in bold and the scalar speed quantities υ_{st} , υ_{s} and υ_{t} are not)

Speed in space, υ_s = Rate of displacement in space $= \dfrac{d}{dt}(x) = \dfrac{dx}{dt}$.

Speed in time, υ_t = Rate of displacement in time $= \dfrac{d}{dt}(ict) = ic$

Speed in spacetime , υ_{st} = Rate of displacement in spacetime $= \dfrac{d}{dt}(s_M) = \dfrac{ds_M}{dt}$

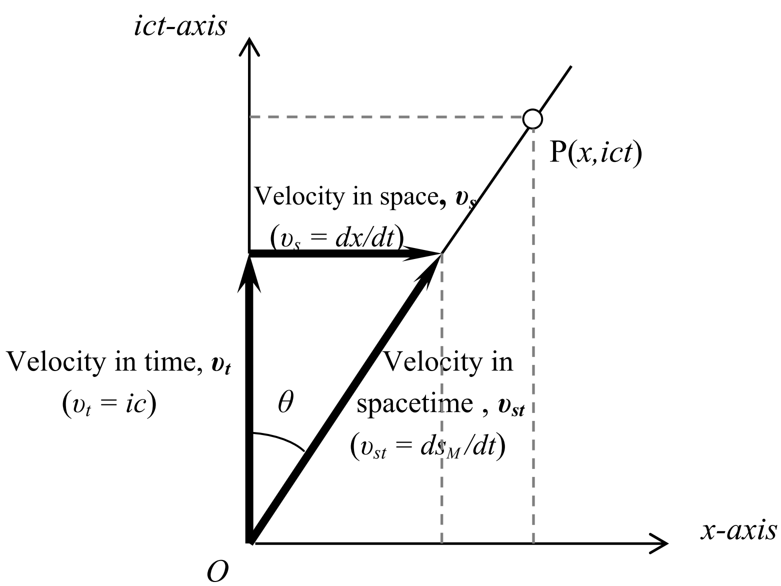

Figure 2: The MST diagram with its velocity vectors.

The vector addition is given by

$$\boldsymbol{\upsilon}_{\mathbf{st}} = \boldsymbol{\upsilon}_{\mathbf{s}} + \boldsymbol{\upsilon}_{\mathbf{t}}$$

$$|\boldsymbol{\upsilon}_{\mathbf{st}}|^2 = |\boldsymbol{\upsilon}_{\mathbf{s}}|^2 + |\boldsymbol{\upsilon}_{\mathbf{t}}|^2$$

$$\upsilon_{\mathrm{st}}^2 = \upsilon_{\mathrm{s}}^2 + \upsilon_{\mathrm{t}}^2$$

$$\left(\frac{ds_M}{dt}\right)^2 = \left(\frac{dx}{dt}\right)^2 + (ic)^2$$

$$(ds_M)^2 = (dx)^2 - c^2(dt)^2$$

This is consistent with the Lorentz equations with the spacetime interval, ds_M, the Lorentz invariant under coordinate transformation. In this diagram, υ_{t} remains invariant as υ_{st} and υ_{s} varies. While υ_s is the coordinate speed υ, the υ_{t} value is a measure of clockrates in the reference frame. Since clockrates in the reference frame is constant, thus υ_{t} which implicitly is directly proportional to clockrates, is a constant in this case. The relationship between angle θ and the magnitudes of $\boldsymbol{\upsilon}_{\mathbf{s}}$ and $\boldsymbol{\upsilon}_{\mathbf{t}}$ is, $\tan\theta = \dfrac{|\boldsymbol{\upsilon}_s|}{|\boldsymbol{\upsilon}_t|} = \dfrac{\upsilon}{ic}$.

The skewing and calibration of the transformed axes in the MST diagram is a consequence of the non-euclidean metric from which it is formulated preserving the light speed invariance c satisfying SR postulate 2. This argument suggests the invariance of the coordinate speed of light restriction be reviewed and calls for a consideration to re-postulate it from broader relativity principles where the light speed invariance is inclusive of this proposed invariance. This motivates us to seek for a viable re-formulation of SR resulting in a convenient euclidean (++++) metric model.

Investigating along this line, a question arises on what this proposed invariance could possibly be? It is implicit from SR that the speed in spacetime of inertial frames is c, suggesting this is inclusive of a photon particle (an inertial frame). Also the clockrates, which by implication is directly proportional to the speed in time υ_{t}, in a photon is zero. This suggests a photon's speed in spacetime c is along the space-axis component corresponding to the coordinate speed υ consistent with SR postulate 2. This reasoning encourages us to conjecture there exist an invariant spacetime speed c in nature for all inertial frames with the invariant coordinate speed c of light as a special case of it.

3.0 A proposed Euclidean spacetime diagram.

In the MST diagram the worldline of a photon's physical path along the *x*-axis is tilted $\frac{\pi}{4}$ radians from the axis. This choice of the light trajectory breaks the euclidean rotation symmetry[1]. A euclidean rotation model implies a real spacetime rotation offering the study of relativistic variations in terms of real space and time. An analysis of *Fig 2* leads us to suspect the inherent non-euclidean rotation limitation in the diagram may be due to the choice of coordinate time, *t*, to represent the speed in time υ_t which is invariant as υ_s (= υ) varies . This argument suggests a geometrical approach be considered where υ_t varies with υ_s in our quest towards formulating a viable euclidean model.

In assigning a time to study a moving frame, besides the coordinate time, *t*, which is with respect to clock readings in the rest frame, at least another option is available, the proper time, τ, which is with respect to clock readings in the moving frame. The option is purely by choice and the observer is privileged to use either. Since SR implies the proper time rates varies with velocity υ (= υ_s), this choice appears to correspond with our argument. We shall adopt this choice of assigning time for the moving frame and represent its υ_t value in terms of proper time, τ. For the vertical time-axis to measure analogous to a length displacement, τ is conveniently multiplied by *c* and with that, the coordinate position of the displacements in space and time of a particle P is ($x,\ c\tau$). As done earlier, by transforming into its rate of displacement in spacetime, space and time, the diagram is presented in terms of velocity vectors.

Similar to *Fig 2*, the velocity vector addition in this proposed alternative diagram is

$$\boldsymbol{\upsilon}_{\mathbf{st}} = \boldsymbol{\upsilon}_{\mathbf{s}} + \boldsymbol{\upsilon}_{\mathbf{t}} \quad \text{...........} \quad \textit{Eqn 1}$$

From postulate 1 (same as in SR), that natural laws in any inertial frame is invariant implies observations between the frames are co-variant. We next raise our conjecture status and put forward <u>postulate 2: Relative to an observer, the speed in spacetime of any inertial frame is an invariant *c*</u>. From this postulate, the magnitude of $\boldsymbol{\upsilon}_{\mathbf{st}}$ is an invariant *c*, given by $|\boldsymbol{\upsilon}_{\mathbf{st}}| = \upsilon_{st} = c$.

The scalar expression of the vector addition in *Eqn 1* is

$$c^2 = \upsilon_s^2 + \upsilon_t^2 \quad \text{...........} \quad \textit{Eqn 2}$$

From *Eqn 2,* the velocity vector diagram in *Fig 3* is governed by the functions of a circle. We will call it as the Euclidean space-time (EST) diagram.

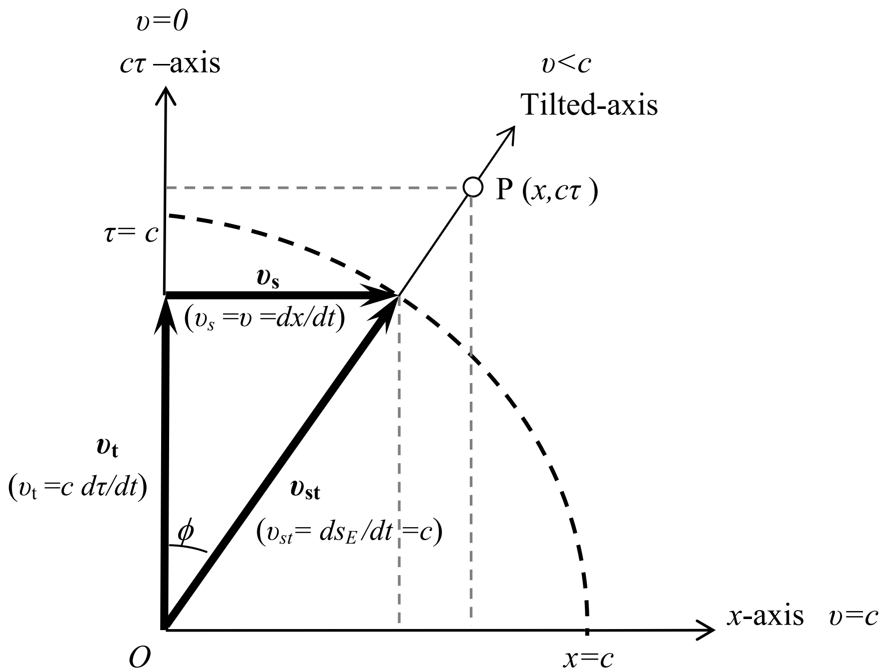

Figure 3: The EST diagram with its velocity vectors.

For a particle P with $\upsilon = 0$, its worldline angular tilt $\phi = 0$ with its $\boldsymbol{\upsilon}_{\mathbf{st}}$ vector along the vertical time-axis. For this case, $|\boldsymbol{\upsilon}_{st}| = |\boldsymbol{\upsilon}_t| = c$ which represents clockrates in a rest frame. For $\upsilon > 0$, the $\boldsymbol{\upsilon}_{st}$ vector tilts from the time-axis and reaches the x-axis when $\upsilon = c$ at $\phi = \pi/2$. From *Eqn 2*, as υ_s varies, the variation in υ_t which is proportional to clockrates in the moving[1] frame preserves the constancy of the spacetime speed c to satisfy postulate 2. In this diagram, the terms in both $\boldsymbol{\upsilon}_{\mathbf{s}}$ and $\boldsymbol{\upsilon}_{\mathbf{t}}$ vectors are real and the spacetime rotation ϕ is real.

The relationship between ϕ, $\boldsymbol{\upsilon}_{\mathbf{s}}$ and $\boldsymbol{\upsilon}_{\mathbf{st}}$ is $\sin\phi = \dfrac{|\boldsymbol{\upsilon}_s|}{|\boldsymbol{\upsilon}_{st}|}$

Since $|\boldsymbol{\upsilon}_s| = \upsilon$ and $|\boldsymbol{\upsilon}_{st}| = c$, $\sin\phi = \dfrac{\upsilon}{c}$ ……… *Eqn 3*

Also from the diagram, $\cos\phi = \dfrac{\upsilon_t}{c}$. Substituting these υ and υ_t values in terms of ϕ, *eqn 2* is the trigonometric identity $1 = \sin^2\phi + \cos^2\phi$. It is of interest that the relationship $\sin\phi = \upsilon/c$ also appears in *Brehme diagram*[2] and *Loedel diagram*[3] formulated to show the relativistic effects from a euclidean rotation analogy.

[1] Since all bodies have a universal speed c in spacetime, ‘moving’ and ‘rest’ hereon refers to its speed in space, υ_s , which is the same as the coordinate speed υ.

Again for this case, $|\boldsymbol{\upsilon_{st}}|$, $|\boldsymbol{\upsilon_s}|$ and $|\boldsymbol{\upsilon_t}|$ represents the speed in spacetime υ_{st} , speed in space υ_s (= υ) and speed in time υ_t .

Speed in space, υ_s = Rate of displacement in space $= \frac{d}{dt}(x) = \frac{dx}{dt}$.

Speed in time, υ_t = Rate of displacement in time $= \frac{d}{dt}(c\tau) = c\,\frac{d\tau}{dt}$

Speed in spacetime , υ_{st} = Rate of displacement in spacetime $= \frac{d}{dt}(s_E) = \frac{ds_E}{dt} = c$

The subscript E in s_E denotes this displacement is in Euclidean spacetime.

The axes of the velocity vector diagram in *Fig 3* corresponds with the 4th dimensional 'velocity in time' $c\frac{d\tau}{dt}$ - axis and the 'velocity in space' $\frac{dx}{dt}$ - axis circular geometry[4] conceptualized on the idea that matter and energy always move at light velocity.

For convenience υ_s is restricted to the x-axis. For any direction along the x-y-z-axes of space,

$$\upsilon_s = \sqrt{\upsilon_x^{\,2} + \upsilon_y^{\,2} + \upsilon_z^{\,2}}$$

$$\upsilon_s = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} \quad \text{.....................} \quad \textit{Eqn 4}$$

Substituting υ_t and υ_s into *Eqn 2.*

$$c^2 = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2 + \left(c\,\frac{d\tau}{dt}\right)^2 \quad \text{..................} \quad \textit{Eqn 5}$$

All the terms are speed parameters with the invariant spacetime speed c on one side and its 4-speed components on the other side. From *eqn 5*, the proper time, $\tau = \int \frac{dt}{\gamma}$, where γ is the Lorentz invariant consistent with proper time τ definition in the mathematical formalism of special relativity.

Re-arranging *eqn 5*,

$$(c\,dt)^2 = (dx)^2 + (dy)^2 + (dz)^2 + (c\,d\tau)^2 \quad \text{..........} \quad \textit{Eqn 6}$$

which is equivalent to the Euclidean form representation of the Minkowski metric. This (++++) metric corresponds with the Euclidean re-formulation of relativistic dynamics by *Montanus*[5] and *Gersten*[6] .
Substituting $ds_E = c\,dt$, *eqn 6* expressed in terms of the displacement in spacetime which is in a real plane is

$$(ds_E)^2 = (dx)^2 + (dy)^2 + (dz)^2 + (c\,d\tau)^2 \text{, the Euclidean metric.}$$

If the Lorentz invariant $ds_M = icd\tau$ is substituted into *eqn 6* instead, the displacement in spacetime which in this case is in a complex plane is

$$(ds_M)^2 = (dx)^2 + (dy)^2 + (dz)^2 - (c\,dt)^2 \text{, the Minkowski metric.}$$

Although *Fig 3* only shows for positive x and τ, the worldlines can be drawn to include negative values for displacement against the x-axis direction and negative values of proper time τ by extending into its past trajectory. In this EST diagram, the worldline of a photon (light) particle corresponds with its physical path along the x-axis. The photon worldlines for both directions is represented by a common line, the x-axis, instead of as two perpendicular lines in the MST diagram each tilted from the x-axis at an angle $\pi/4$ radian. In this diagram all regions are time-like with the light-like path along the x-axis, thus the undefined space-like regions are avoided compared to the MST diagram's cone representation. The EST diagram is consistent with the requirement to study relativistic variations in terms of a euclidean rotation. As recently proposed[7], we will call our euclidean re-formulation of special relativity, as Euclidean special relativity, ESR.

4.0 Applying the EST diagram.

(a) Relativistic variation.

In the velocity vector EST diagram, corresponding to the displacement sign change, υ is positive along the x-axis direction and negative against it. As υ ranges from 0 to c, ϕ ranges from 0 to $\frac{\pi}{2}$ and as υ ranges from 0 to $(-c)$, ϕ ranges from 0 to $(-\frac{\pi}{2})$. A rest frame's $\boldsymbol{\upsilon}_{\mathbf{st}}$ vector is along the vertical time-axis and the magnitude $|\boldsymbol{\upsilon}_{\mathbf{t}}|_{\text{Rest}} = \upsilon_{t\ \text{Rest}}$ is equal to the invariant spacetime speed c. A moving frame's $\boldsymbol{\upsilon}_{\mathbf{st}}$ vector is tilted at an angle ϕ from the reference vertical time –axis and the $|\boldsymbol{\upsilon}_{\mathbf{t}}|_{\text{Moving}} = \upsilon_{t\ \text{Moving}}$ is obtained by resolving the invariant spacetime speed c along the axis as $c\cos\phi$. Since υ_t is proportional to clockrates, the clockrates ratio is equal to the υ_t ratio between a moving and rest frame.

$$\frac{\upsilon_{t\,Moving}}{\upsilon_{t\,Rest}} = \frac{c \cos\phi}{c} = \cos\phi.$$

The time dilation (*TD*), is the clockrates ratio between a rest and moving clock and is the inverse of the above ratio. Thus $TD = \sec\phi$. From trigonometric identities and substituting $\sin\phi = \upsilon / c$ from *eqn 3*, $TD = \dfrac{1}{\sqrt{1-\dfrac{\upsilon^2}{c^2}}}$

Also the angular tilt ϕ of the moving frame's worldline from the reference rest frame's worldline along the vertical-axis, causes a length variation effect. A rest rod parallel to the *x*-axis measured as L_{Rest} is tilted at an angle ϕ when moving. Resolving for this angular tilt, the observed length of the rod when moving, $L_{Moving} = L_{Rest} \cos\phi$.

The length contraction ratio = $\dfrac{L_{moving}}{L_{rest}} = \dfrac{L_{rest} \times \cos\phi}{L_{rest}} = \cos\phi = \sqrt{1-\dfrac{\upsilon^2}{c^2}}$.

Investigating photon emissions received from a moving body[8], the length variation is expressed in terms of an angular rotation = $\cos^{-1}\sqrt{1-\dfrac{\upsilon^2}{c^2}}$ corresponding with ϕ in our model. Postulating[9] objects flow at a constant 4th speed c in time, the variations are shown by assuming $\upsilon = c \cos\Theta$, where Θ is the complementary angle of ϕ . A euclidean approach with a velocity in time dimension analogous to velocity in spatial dimensions linked by a speed c has also been proposed[10]. Investigating on the spherical wavefront of a light pulse[11], a 4-coordinate manifold of SR has been modeled governed by the functions of a circle.

For $-\dfrac{\pi}{2} < \phi < 0$, corresponding to negative velocity υ range, $\sin(-\phi) = -\sin\phi$ consistent with the velocity sign change. For the whole ϕ range, $\cos\phi$ and $\sec\phi$ remains positive consistent with time and space variations as independent of either direction. When $\upsilon << c$, $|\boldsymbol{\upsilon}_t| \approx |\boldsymbol{\upsilon}_{st}| = c$ and the constancy of $\upsilon_{st} = c$ (universal velocity) reduces to the constancy of $\upsilon_t = c$ (universal time). For this case, $\phi \approx 0$ and $\cos\phi \approx 1$ and $\sec\phi \approx 1$, showing the relativistic variations are negligible.

Applying trigonometric identities from $\sin\phi = \upsilon/c$, $\sec\phi = \dfrac{1}{\sqrt{1-\dfrac{\upsilon^2}{c^2}}}$ and $\tan\phi = \dfrac{\upsilon/c}{\sqrt{1-\dfrac{\upsilon^2}{c^2}}}$

Substituting into the 1st and 4th equation coefficients, the Lorentz transformations are re-expressed in terms of ϕ, in trigonometric form as

$x_1' = (\sec\phi)\, x_1 - (\tan\phi)\, x_4$ …… 1st Eqn

$x_4' = -(\tan\phi)\, x_1 + (\sec\phi)\, x_4$ ….. 4th Eqn

compared to the hyperbolic form in terms of rapidity α in SR where $\tanh\alpha = \upsilon/c$, as

$x_1' = (\cosh\alpha)\, x_1 - (\sinh\alpha)\, x_4$ …… 1st Eqn

$x_4' = -(\sinh\alpha)\, x_1 + (\cosh\alpha)\, x_4$ ….. 4th Eqn

Comparing $\upsilon/c = \tanh\alpha = \sin\phi$ with the Gudermann (Φ) transformation[12], we obtain $\sin(\Phi(\alpha)) = \tanh(\alpha)$, giving an intimate relationship between the angles as $\phi = \Phi(\alpha)$. These angular velocity parameters ϕ and α are the variables in the two schemes approach in deriving the Lorentz boost[13]. A proposal for a trigonometric spacetime formalism in relativistic investigations has also been offered[14].

Postulating $\sin\phi = \frac{\upsilon}{c}$, the Lorentz equations are expressed by *Majernik*[15] in trigonometric form as a function of ϕ. He states,*" The above trigonometric form of the Lorentz transformation is interesting not only from a formal point of view, but may serve as a starting point for expressing other relativistic quantities in terms of trigonometric functions and for the formulation of relativistic relations in the language of trigonometry."* Although a physical interpretation for ϕ is not shown, investigating[16] on photons received from a moving body, *Majernik's* space-time angle ϕ is shown to be an aberration angle which corresponds with our model.

(b) Coordinate transformation.

In the EST diagram, the reference displacement coordinate axes are the vertical time $c\tau$ –axis and horizontal space x-axis. The transformed axes are the time $c\tau'$-axis and space x'-axis with both tilted from the reference axes at an angle ϕ in the same direction maintaining orthogonolity. The transformed axes coordinate units are its intersections with the circular function curves corresponding to the euclidean metric transformation equation. The need for calibration is avoided and the square grids remains unchanged under transformation. The reference and transformed coordinate positions are (x,τ) and (x',τ') respectively. When $\upsilon=0$, $\phi=0$ and both pair of axes coincides with $x = x'$ and $\tau = \tau'$.

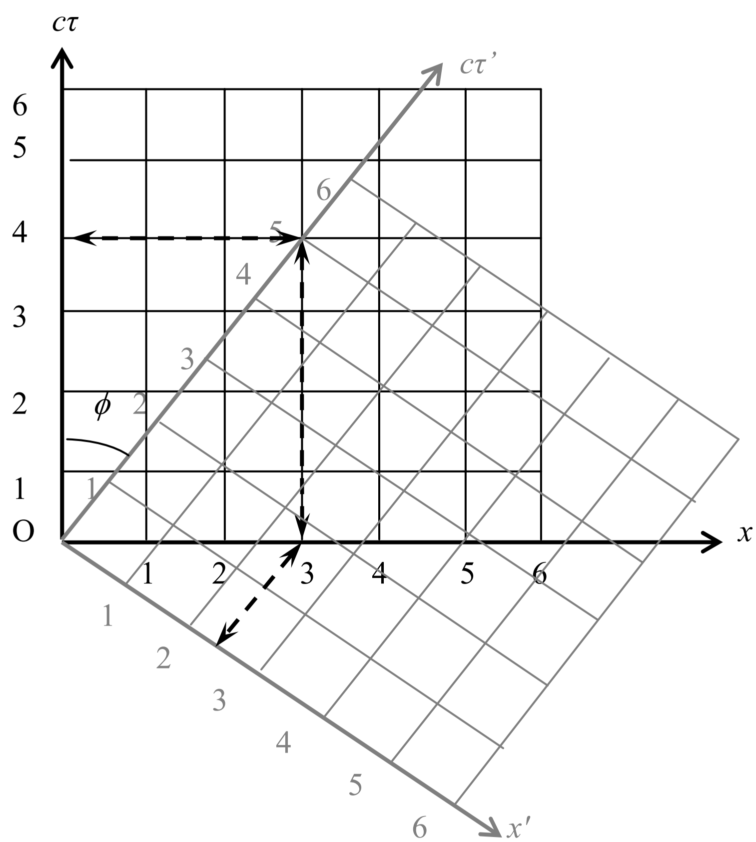

Figure 4 : The transformed coordinates in EST diagram.

Fig 4 shows the transformed coordinates for the case $\upsilon=0.6$ (taking $c=1$). Lines parallel to the space and time axes are lines of simultaneity and equidistance with respect to the frames. The intersection points $\tau=4$ (proper time) with $\tau'=5$ (coordinate time) and $x=3$ with $x'=2.4$ are consistent with Lorentz transformations. The transformed $c\tau'$ time-axis tilts towards the horizontal-axis as υ varies corresponding with the angular tilt ϕ of the transformed time-axis in the euclidean modeling[17] given by $\sin\phi = \upsilon/c$.

The derived relativistic variations in ESR is a consequence of preserving the spacetime speed invariance compared to the coordinate speed invariance of light in SR. On the Lorentz case, *Leblond*[18] states, "…. *This is the point of view from which I intend to criticize on the overemphasized role of the speed of light in the foundations of special relativity*" and further adds *" The Lorentz case is characterized by a parameter with the dimensions of a velocity which* *is a universal constant associated with the very structure of space-time".* It is of interest the work by [8], [11] and [16] studying on photons received from a moving body based on the constancy of light velocity *c* produces results consistent with the EST diagram. The de Broglie waves too are expressed in terms of ϕ in trigonometric form[15] with the relativistic variations correlated graphically by a single diagram. Also based on a wave field system[19], a diagram corresponding with the EST diagram has been presented.

5.0 Comparing the MST and EST diagram.

In both diagrams, the world is modeled as 4-spacetime with 3-space and a 4th-time dimension. Both spacetime angles θ and ϕ uniquely varies with velocity υ and are zero at $\upsilon=0$ with its rotation equivalent to an acceleration. In both, a singularity is approached as velocity approaches a limiting value *c*. For the special case of velocities much less than *c*, the transformation reduces to Galilean transformation to the 1st order of approximation.

The MST diagram is a geometrical extraction of the non-euclidean transformation metric derived from SR postulates. The negative sign in the metric requires the presence of *i* along the vertical time-axis in its geometrical formulation. As a result a particle's trajectory in spacetime is in a complex plane tilted at an unreal spacetime angle θ. The equations are governed by the functions of a hyperbola expressed in terms of rapidity α. The transformed axes rotates in opposite directions as velocity υ varies resulting in skewing and requiring a calibration of the coordinate units.

In comparison, the EST diagram is a euclidean metric formulation based on ESR postulates. In this diagram, the vertical coordinate time *t*-axis is replaced by the proper time τ-axis thus avoiding the presence of *i* along it. As a result a particle's trajectory in spacetime is in a real plane tilted at a real spacetime angle ϕ. The equations are governed by the functions of a circle expressed in trigonometric form in terms of ϕ. The transformed axes rotates in the same direction as velocity υ varies maintaining orthogonality and avoiding the need for its unit calibration. The consistency of the EST diagram with the Lorentz transformations offers a new geometrical tool to investigate relativistic observations and encourages considerations on its viability to serve as a convenient alternative to the MST diagram.

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