APPENDIX TO PAPER 6

THE De BROGLIE WAVES CONNECTION WITH THE EUCLIDEAN MODEL.

(Paper 6: *The advantage of euclidean interpretation of relativity*
www.euclideanrelativity.net , S. Kanagaraj, 19 Feb 2010).

Using the mathematical postulate, $\upsilon = c \sin \phi$, *Majernik* has shown the connection between important relativistic quantities for particles with de Broglie waves as follows:-

- **de Broglie phase waves**

The x and t components of the 4-velocity are $\upsilon_1 = \gamma \upsilon$ and $\upsilon_4 = \gamma c$. Substituting $\upsilon = c \sin \phi$ and $\gamma = \sec \phi$, then $\upsilon_1 = c \tan \phi$ and $\upsilon_4 = c \sec \phi$. Since $\sec^2\phi - \tan^2\phi = 1$, thus $\upsilon_4{}^2 - \upsilon_1{}^2 = c^2$, showing that the 4-velocity vector magnitude is invariant under Lorentz transformation. Also

$$E = \gamma m_o c^2 = m_o c^2 (\sec\phi)$$

and

$$p = \gamma m_o \upsilon = m_o c (\tan \phi)$$

Again using the above trigonometric relationship, $E^2 - p^2c^2 = (m_o c^2)^2$

Using the Einstein and de Broglie 'connections'

$E = h\nu$ …….. [1]
and $p = h/\lambda$ …….. [2]

From [2], the wavelength associated with a particle of rest mass m_o may be written as

$$\lambda = (h/m_o c)(\cot \phi) = \lambda_o (\cot \phi) \text{ …….. [3]}$$

where $\lambda_o = h/m_o c$ is the Compton wavelength of m_o.

From [1], the frequency ν of this phase wave is

$$\nu = mc^2/h = (m_o c^2/h) (\sec \phi) = \nu_o (\sec \phi) \text{ …… [4]}$$

where a 'Compton frequency' $\nu_o = m_o c^2/h$ is defined in analogy with the Compton wavelength λ_o.

Noting that $\lambda_o \nu_o = c$, the phase velocity υ_{ph} of the de Broglie phase wave may be written as

$$\upsilon_{ph} = \lambda \nu = c\ (cosec\ \phi) \quad \ldots\ldots\ldots [5]$$

For completeness, the group velocity of the phase wave is also calculated,

$$\upsilon_{gr} = \frac{d\nu}{dk} \quad \ldots\ldots\ldots [6],$$ where k is the wavenumber $k = 1/\lambda$.

By using [3], k can be expressed trigonometrically as

$$k = (tan\ \phi)/\lambda_o = k_o\ (tan\ \phi) \quad \ldots. [7],$$ where $k_o = 1/\lambda_o$.

Since both ν and k depend on the angle ϕ, from [4] and [7],
we can write

$$\upsilon_{gr} = \frac{(d\nu/d\phi)}{(dk/d\phi)} = c\ (sin\ \phi) \quad \ldots\ldots [8]$$

This is identical to the mathematical postulate $\upsilon = c\ (sin\ \phi)$ because the group velocity of the de Broglie waves corresponds to particle velocity υ.

The internal consistency of the trigonometric approach is further verified by multiplying υ_{ph} and υ_{gr} (eqns [5] & [8]) which should yield c^2.

$$\upsilon_{ph}\,\upsilon_{gr} = c\ (cosec\ \phi)\ c\ (sin\ \phi) = c^2.$$

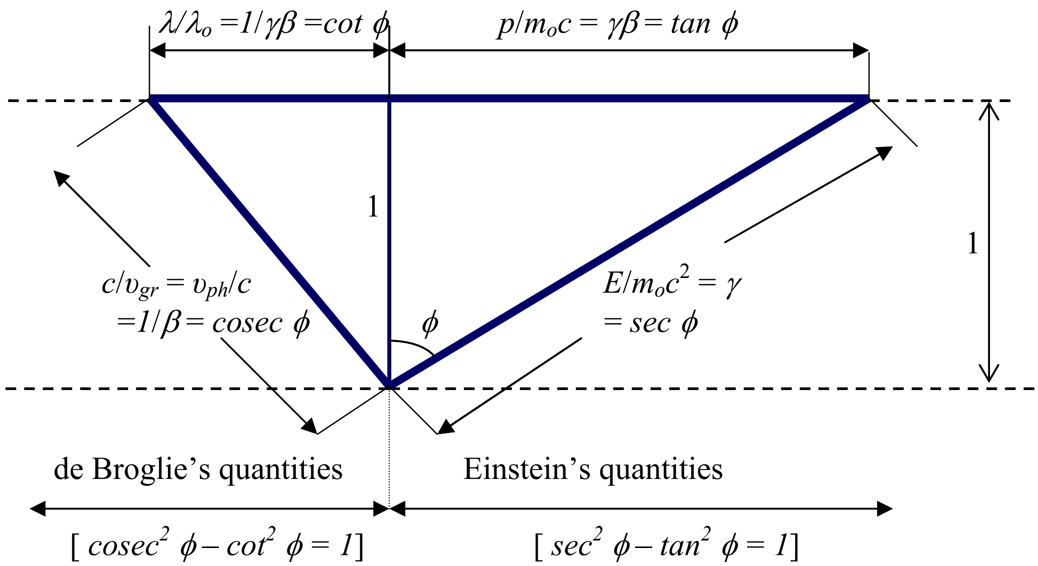
Similarly as in the case of particle dynamics which yields $E^2 - p^2c^2 = (m_oc^2)^2$, the trigonometric identity $sec^2\ \phi - tan^2\ \phi = 1$, yields the relativistic dispersion relation for de Broglie waves,

$$(\nu/\nu_o)^2 - (k/k_o)^2 = 1$$

or $\nu^2 = c^2(k_o^2 + k^2)$

• Graphical representation

The Einstein's and de Broglie's quantities are corelated graphically by a single diagram.



The simple graphical technique in the above diagram is an extension of the frequently used "mnemonic" right triangle with E as hypotenuse and pc and $m_o c^2$ as its sides. The diagram shows the correlation of the relativistic particle quantities with the corresponding de Broglie quantities. From trigonometric identities and/or Pythagorean theorem, a number of basic relationships may be derived directly from this diagram.

Not shown in the figure are the components of the 4-velocity, υ_1 & υ_4. Associating these quantities with the appropriate segments of the right angled triangle, the invariance relationship $\upsilon_4{}^2 - \upsilon_1{}^2 = c^2$ can be obtained directly from the figure.

Note :
The data shows the orientation angle ϕ in the euclidean diagram has an intimate relationship with both the above quantities. In *Paper 6*, the connection between the (a) velocity in space υ_s and velocity in time υ_t and (b) momentum p and energy E of a frame is given by $sin^2\ \phi + cos^2\ \phi = 1$ and $sec^2\ \phi - tan^2\ \phi = 1$ respectively. The connection between λ/λ_o and $c/\upsilon_{gr} = \upsilon_{ph}/c$ is again given by the trigonometric identity $cosec^2\ \phi - cot^2\ \phi = 1$. Alternatively these connections may be expressed as a function of the space-time angle θ as $cos^2\ \theta + sin^2\ \theta = 1$; $cosec^2\ \theta - cot^2\ \theta = 1$ and $sec^2\ \theta - tan^2\ \theta = 1$ respectively.

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