A EUCLIDEAN ALTERNATIVE TO MINKOWSKI SPACETIME DIAGRAM.

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Abstract

By re-interpreting the special relativity (SR) postulates based on Euclidean 4-space-time , a geometrical model is formulated called the Euclidean Space-Time (EST) diagram. The EST diagram is a velocity vector model with a real time parameter along the 4th dimension and consequently has a (++++) metric. The model is governed by a circular function geometry and the relativistic variations are expressed in trigonometric form as a function of a real spacetime angle θ (or alternatively an orientation angle ϕ) that uniquely corresponds to velocity v. Its possible usage as a convenient alternative to Minkowski diagram to investigate the Lorentz transformation is discussed.

Keywords: Special relativity, Euclidean 4-space-time, Lorentz transformation.

1.1 INTRODUCTION.

The fundamentals of physical phenomena can generally be interpreted in terms of simple operations of geometry and has often been applied to simplify our understanding of it. In Galilean relativity, based on the premise that mechanical laws remains the same to an observer, the principles of mechanics were formulated. With the introduction of special relativity (SR), mechanics and optics were incorporated into the relativity framework by postulating natural laws remains the same in any inertial reference frame and that light velocity remains the same *(c)* independent of the motion status of the source. The Lorentz transformation equations were formulated studying two (2) inertial frames O and O' moving along the *x* and *x'*-axis using position coordinates in similarity to that as done under the Galilean transformation but with the added requirement it satisfies the SR postulates. For the special case of velocities much less than *c*, the Lorentz transformation reduces to Galilean transformation to the 1st order of approximation.

The wavefront of the invariant velocity *c* of a propagating light pulse with reference to the coordinate system of frame O is $x^2 + y^2 + z^2 - c^2 t^2 = 0$ and of frame O' is $x'^2 + y'^2 + z'^2 - c^2 t'^2 = 0$. The geometrical model formulated from SR based on this position coordinates is the Minkowski space-time (ST) diagram. The components of the 4-vector for displacement in 4-dimensional (4D) space-time is $ds^2 = dx^2 + dy^2 + dz^2 - c^2 dt^2$, where *ds* is the Lorentz invariant. The 4th dimensional-axis in the ST diagram is imaginary and thus has a non-Euclidean (+++ −) metric. A moving body is studied in terms of a complex angular rotation in space-time with the Lorentz transformation equations expressed as a hyperbolic function of an angular

velocity parameter, rapidity α. This motivates us to seek a convenient alternative Euclidean (+ + + +) metric model with the relativistic equations expressed as simple trigonometric functions of an angular velocity parameter derivable from this geometry.

1.2 THE CONCEPTUAL SPACETIME GEOMETRY.

To formulate the conceptual geometry, we put forward the 1st postulate.

Postulate 1: Relative to an observer in any inertial frame, natural laws remains the same.

This postulate 1 is the same as in SR. It implies for the case of 2 inertial frames O & O' at rest relative to each, the space and time intervals in the frames for both observers O & O' remains the same at its 'proper' values. When O' is moving, the intervals are not 'proper' relative to observer O with the observations being co-variant. If instead of only 2 inertial frames O & O', if each is made up of a group of inertial frames O_1, O_2, O_3, O_4, O_5, O_6 & O_1', O_2', O_3', O_4', O_5', O_6' located orthogonally moving at a uniform velocity v (*Fig 1*), then observations of the variations between any 2 frames in different groups also remains exactly the same.
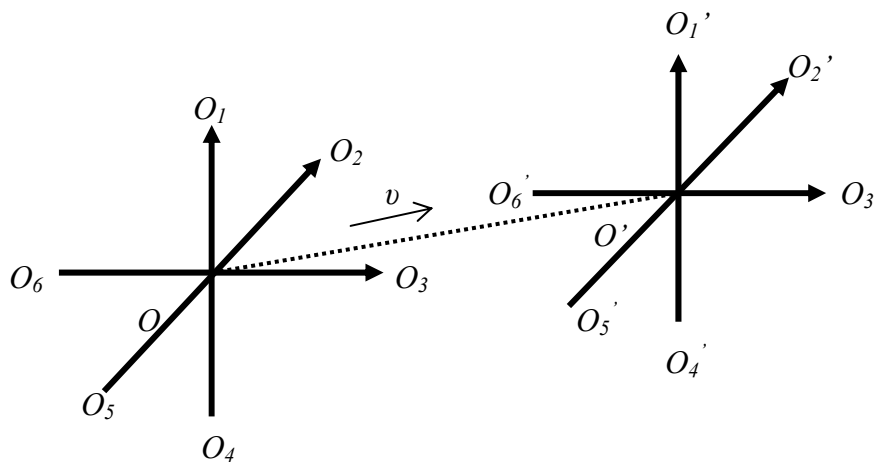

Fig 1: 2 groups of inertial frames O and O' moving uniformly at velocity v.

We conceptualize the 'rest' and 'moving' groups of frames are in Euclidean spaces relative to observers within a group and are rotated in 4-space-time with observations being co-variant. In our approach a frame is investigated not only in relation to its rate of displacement in space (velocity in space, v_s) conventionally called its velocity v but also in relation to its rate of displacement in time (velocity in time, v_t). This suggests that an inertial frame's rate of displacement in spacetime (its 'velocity in spacetime'), be represented by a velocity spacetime vector $\boldsymbol{v_{st}}$ with its 2 components as the velocity space vector $\boldsymbol{v_s}$ and velocity time vector $\boldsymbol{v_t}$. For convenience we fix the direction of vector $\boldsymbol{v_s}$ (representing the conventional velocity) along the x_1-axis and with that the only axes of interest are the x_1 and x_4 axes. This implies the other component of the frame's $\boldsymbol{v_{st}}$ vector be represented by a velocity time vector $\boldsymbol{v_t}$ normal to the $\boldsymbol{v_s}$

vector. The 'velocity in time' v_t of a frame is a measure of its clockrates. *Fig 2* shows this conceptual geometrical model.

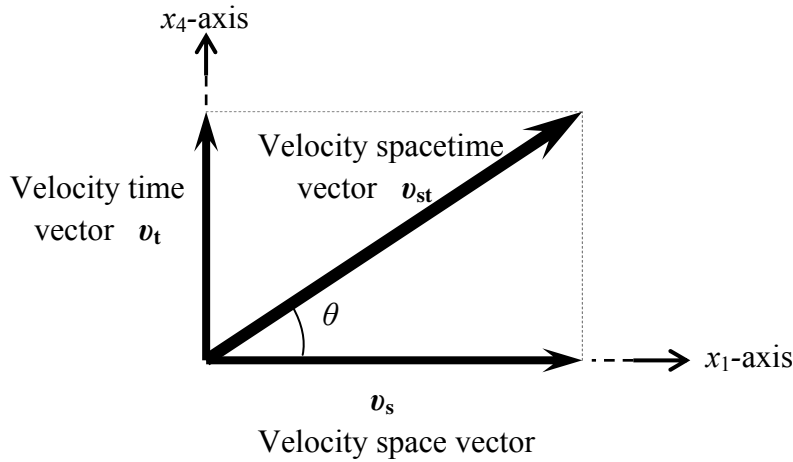**Fig 2: The conceptual spacetime model.**

The magnitudes of the velocity vectors $\boldsymbol{v_s}$, $\boldsymbol{v_t}$ and $\boldsymbol{v_{st}}$ (in bold) represent the speeds v_s (=v), v_t and v_{st} (scalar quantities, not in bold) respectively. We will call the angular inclination θ of a frame's velocity in spacetime as the spacetime angle. Relative to an observer, v_s and v_t are real thus both vectors $\boldsymbol{v_s}$ and $\boldsymbol{v_t}$ are real and consequently v_{st} and θ are also real.

Expressing the velocity vector addition in *Fig 2* ,

$$\boldsymbol{v_{st}} \; = \; \boldsymbol{v_s} \; + \; \boldsymbol{v_t} \quad \text{......... } \textbf{\textit{Eqn 1}}$$

Inertial frames within a group are in the same Euclidean space with those in other groups in Euclidean spaces that are rotated from each other. We next proceed with the formulation of the spacetime geometrical model.

1.3 THE SPACETIME GEOMETRY – THE EUCLIDEAN SPACE-TIME (EST) DIAGRAM.

To formulate the geometry we put forward the next postulate,

Postulate 2: Relative to an observer the velocity in spacetime of any inertial frame remains the same at *c*.

Applying this postulate , the magnitude of $\boldsymbol{v_{st}}$ is an invariant *c*. *Linden* [1] has proposed that velocity in time is a physical property in the time dimension analogous to velocity in spatial

dimensions (i.e. velocity in space) linked by a constant velocity c. Since $\left|\boldsymbol{v_{st}}\right| = v_{st} = c$ by postulate , the scalar expression of the vector addition in *Eqn 1* is

$$c^2 = v_s{}^2 + v_t{}^2 \quad \text{............} \quad \textbf{\textit{Eqn 2}}$$

From *Eqn 2,* the spacetime geometry (*Fig 3*) is governed by the functions of a circle. We will call it as the Euclidean Space-Time (EST) diagram.
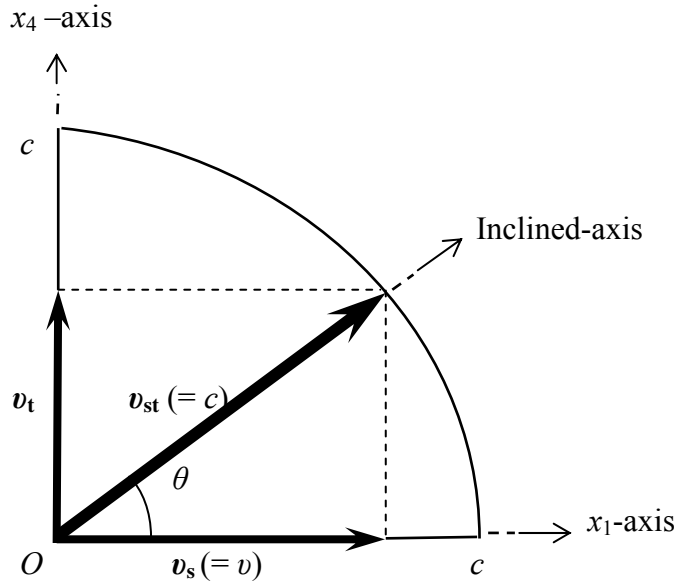

Figure 3: The circular function EST diagram.

Inertial frames always move at velocity in spacetime c along a trajectory at an angle θ that uniquely corresponds to its velocity v. The magnitudes of the vector components of $\boldsymbol{v_{st}}$ are $\left|\boldsymbol{v_s}\right| = v_s = v$ and $\left|\boldsymbol{v_t}\right| = v_t$. From *Fig 3*, v_s (= v) cannot exceed c consistent with causality requirements. For the case when v_s (= v) = 0, $\boldsymbol{v_{st}}$ is exactly along the x_4-axis with $\theta = \dfrac{\pi}{2}$ and $v_t = c$. Thus a 'rest' frame has $v_t = c$ along the x_4-axis which represents the proper clockrates.

Since v_t is proportional to clockrates, the clockrates ratio between a moving and rest frame is

$$= \frac{v_t \text{ of moving frame}}{v_t \text{ of rest frame}}$$

$$= \frac{v_t}{c} \quad \text{...................} \quad \textbf{\textit{Eqn 3}}$$

If the clock readings in the moving and reference rest frame is denoted as τ and t respectively, the ratio of the difference in the clock readings between the moving and rest frame is

$$= \frac{\tau_2 - \tau_1}{t_2 - t_1}$$

$$= \frac{\Delta \tau}{\Delta t} \quad \text{................} \quad \textbf{\textit{Eqn 4}}$$

The ratios of the moving and rest frame's clockrates and the difference in their clock readings are the same. Equating *Eqn 3* and *Eqn 4*, $v_t = c \dfrac{\Delta \tau}{\Delta t}$.

Expressed in differential form in the limit $\Delta t \to 0$,

$$v_t = c \frac{d\tau}{dt} \quad \text{............} \quad \textbf{\textit{Eqn 5}}.$$

For convenience, we fixed a frame's velocity in space along the x_1-axis. For the case in any direction, $v_s = v = \dfrac{dx}{dt}$. Based on the idea that all forms of matter and energy move at velocity of light, *Bradford* [2] has conceptualized a circular geometry with the 4th -dimensional 'velocity in time' axis as $c\dfrac{d\tau}{dt}$ and the 'velocity in space' axis as $\dfrac{dx}{dt}$ $(= v)$ consistent with our EST diagram.

The velocity in space v_s for any direction in the x_1-x_2-x_3-axes of space is

$$v_s = \sqrt{v_{x1}^2 + v_{x2}^2 + v_{x3}^2}$$

$$v_s = \sqrt{\left(\frac{dx_1}{dt}\right)^2 + \left(\frac{dx_2}{dt}\right)^2 + \left(\frac{dx_3}{dt}\right)^2} \quad \text{....................} \quad \textbf{\textit{Eqn 6}}$$

Substituting v_t and v_s from *Eqn 5* and *Eqn 6* into *Eqn 2*.

$$c^2 = \left(\frac{dx_1}{dt}\right)^2 + \left(\frac{dx_2}{dt}\right)^2 + \left(\frac{dx_3}{dt}\right)^2 + \left(c\frac{d\tau}{dt}\right)^2 \quad \text{....................} \quad \textbf{\textit{Eqn 7}}$$

Both v_s (= v) and v_t constitute an observer's physical reality, therefore the 2 axes in the EST diagram are real. All the terms in *Eqn 7* are motion parameters with the invariant velocity in spacetime *c* on the LHS and its 4-vector velocity components on the RHS.

Re-arranging *Eqn 7*,

$$(c\,dt)^2 = (dx_1)^2 + (dx_2)^2 + (dx_3)^2 + (c\,d\tau)^2 \quad \text{...........} \textbf{\textit{Eqn 8}}$$

which is the Euclidean form representation of the Minkowski metric.

Substituting the Lorentz invariant $ds = icd\tau$, into *Eqn 8*,

$$(ds)^2 = (dx_1)^2 + (dx_2)^2 + (dx_3)^2 - (c\,dt)^2 \text{ , the Minkowski metric.}$$

The 2 postulates put forward in our formulation is based upon a Euclidean interpretation of SR. We will call it as Euclidean special relativity [3], ESR, as recently proposed. The model corresponds with the Euclidean re-formulation of relativistic dynamics by *Montanus* [4] and *Gersten* [5] .

Since $v_s = v$, from *Eqn 2*,

$$v_t = \sqrt{c^2 - v^2} \quad \text{............} \textbf{\textit{Eqn 9}}$$

From the EST diagram, $cos\ \theta = \dfrac{v_s}{v_{st}} = \dfrac{v}{c}$. Substituting $v = c\ cos\ \theta$ into *Eqn 9*, $v_t = c\ sin\ \theta$. *Eqn 2* reduces to the trigonometric identity , $1 = cos^2\ \theta + sin^2\ \theta$. Adopting a convention where v is positive (+ve) for receding motion and negative (–ve) for approaching motion, as v ranges from *0* to *c* , θ ranges from $\dfrac{\pi}{2}$ to *0* and as v ranges from *0* to (– *c*) , θ ranges from $\dfrac{\pi}{2}$ to π. The circular function for the whole range is represented by the right quadrant ($\dfrac{\pi}{2} \geq \theta \geq 0$) and left quadrant ($\dfrac{\pi}{2} \leq \theta \leq \pi$) respectively. Since $v = c\ cos\ \theta$, for approaching motion range, the cosine changes from *+ve* to *–ve* and only the right quadrant (*Fig 3*) is needed to represent both directions.

1.4 DERIVING THE RELATIVISTIC TIME AND SPACE VARIATION.

The clockrates ratio of a 'moving' and a 'rest' frame [1] is

$$\frac{v_t \ of \ moving \ frame}{v_t \ of \ rest \ frame \ (= c)} = \frac{\sqrt{c^2 - v^2}}{c} = \sqrt{1 - \frac{v^2}{c^2}}$$

Clockrates are inversely proportional to time dilation (TD), thus

$$TD \ ratio = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad .... \quad \textbf{Eqn 10}$$

When $v \ll c$, $|v_t| \approx |v_{st}| = c$ and the constancy of $v_{st} = c$ (universal velocity) reduces to the constancy of $v_t = c$ (universal time). Substituting $v = c \cos \theta$, the clockrate and TD ratios are $sin \ \theta$ and $cosec \ \theta$ respectively. Due to time dilation, the clock readings τ represents the 'proper' time elapsed in the moving frame. When $v \rightarrow c$, $\theta \rightarrow 0$ and the clockrate ratio, $sin \ \theta \rightarrow 0$ and TD ratio $cosec \ \theta \rightarrow \infty$. When $v \ll c$, $\theta \approx \frac{\pi}{2}$ and the ratios are $sin \ \theta \approx 1$ and $cosec \ \theta \approx 1$.

In the EST diagram, the reference 'rest' (i.e. $v = 0$) frame moves along the x_4-axis at speed c. A 'moving' (i.e. $v \neq 0$) frame is inclined at an angle $(\theta_o - \theta)$ where $\theta_o \ (= \pi/2)$ and θ are the spacetime angles when at 'rest' and 'moving' respectively. If L_{rest} and L_{moving} are the observed lengths of a frame at 'rest' and 'moving' , then $L_{moving} = L_{rest} \times cos \ (\theta_o - \theta)$.

$$space \ variation \ ratio = \frac{L_{moving}}{L_{rest}} = \frac{L_{rest} \times cos \ (\theta_o - \theta)}{L_{rest}} = cos \ (\theta_o - \theta) = cos \left(\frac{\pi}{2} - \theta \right) = sin \ \theta$$

Substituting $cos \ \theta = \frac{v}{c}$ into the trigonometric identity $sin^2 \ \theta + cos^2 \ \theta = 1$,

$$space \ variation \ ratio = \sqrt{1 - \frac{v^2}{c^2}} \quad .... \quad \textbf{Eqn 11}$$

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[1] Since all bodies have a universal velocity c in spacetime, "moving" and "rest" hereon refers to its velocity in space v_s, which is the same as the conventional velocity v.

Similarly, v_t of a moving frame is $c\ cos(\theta_o - \theta)$ and the time variation ratio can alternatively be derived as $c\ cos\ (\theta_o - \theta)\ /c = sin\ \theta = \sqrt{1 - \dfrac{v^2}{c^2}}$. For approaching motion, $\dfrac{\pi}{2} \le \theta \le \pi$ and $sin\ (\pi - \theta) = sin\ \theta$ showing the space and time variation , $sin\ \theta$, remains the same for both directions. When $v << c,\ \theta \approx \theta_o = \dfrac{\pi}{2}$ and $sin\ \theta \approx sin\ \theta_o = 1$. The variations are negligible and it can practically be assumed that bodies move in Euclidean space for this case. As $v \to c,\ \theta \to 0$ and $sin\ \theta \to 0$, implying a body approaches a singularity as its velocity v approaches c.

In the EST diagram, the space and time variation are conveniently modeled as a common spacetime variation with a single-axis, the velocity spacetime vector. The spacetime aberration, $\left(\dfrac{\pi}{2} - \theta\right)$, is the rotation away from the 'proper' time x_4-axis and 'proper' length x_1-axis. *Fig 4* shows these 2 axes as orthogonal with both rotating in the same direction.

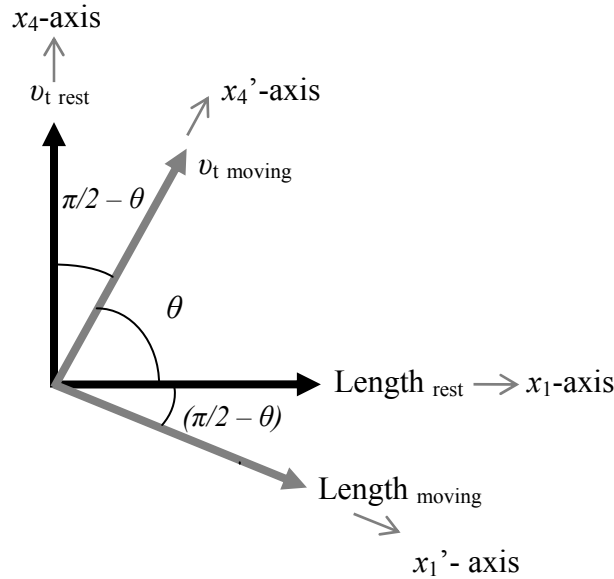**Fig 4: The θ relationship with space and time variations.**

The variation ratio is the projection of the inclined x_1'-axis space and x_4'-axis time onto the 'proper' x_1-axis space and x_4-axis time respectively. The *time variation ratio* $= \dfrac{v_{t\ Moving}}{v_{t\ Rest}} =$

$\dfrac{c\ cos\left(\dfrac{\pi}{2} - \theta\right)}{c}$ $= sin\ \theta$. Also if the observed length of a moving and rest frame is L' and L, the

space variation ratio $= \dfrac{L'}{L} = \dfrac{L\cos\left(\dfrac{\pi}{2}-\theta\right)}{L} = sin\ \theta.$ By postulating [6] objects move in time
(4th speed) with a constant 'flow' c and assuming $v = c\cos\theta$, the variations are shown as $sin\ \theta$ consistent as derived above.

From the EST diagram $cos\ \theta = \dfrac{v}{c}.$ Applying trigonometric identities

$$cosec\ \theta \ = \ \frac{1}{\sqrt{1-\dfrac{v^2}{c^2}}} \quad and \quad cot\ \theta \ = \ \frac{v/c}{\sqrt{1-\dfrac{v^2}{c^2}}}$$

The 1st and 4th equations of the Lorentz transformation written as

$$x_1' = \ \frac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}\ x_1\ -\ \frac{v/c}{\sqrt{1-\dfrac{v^2}{c^2}}}\ x_4 \quad \ldots\ldots\ldots\ 1^{st}\ \text{Eqn}$$

$$x_4' = \ -\ \frac{v/c}{\sqrt{1-\dfrac{v^2}{c^2}}}\ x_1\ +\ \frac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}\ x_4 \quad \ldots\ldots\ldots\ 4^{th}\ \text{Eqn}$$

are re-expressed using the angular velocity parameter, θ, in trigonometric form as

$x_1' = (cosec\ \theta)\ x_1 - (cot\ \theta)\ x_4$ ……. 1st Eqn

$x_4' = -(cot\ \theta)\ x_1 + (cosec\ \theta)\ x_4$ …… 4th Eqn

as compared to expressing it in hyperbolic form presently using SR with the angular velocity parameter rapidity α as

$x_1' = (cosh\ \alpha)\ x_1 - (sinh\ \alpha)\ x_4$ …… 1st Eqn

$x_4' = -(sinh\ \alpha)\ x_1 + (cosh\ \alpha)\ x_4$ ….. 4th Eqn

We derived the relativistic equations consistent with the Lorentz transformation applying relativity principles based only on the invariance of the relations between many equivalent reference frames obeying a group law. *Leblond* [7] has derived the Lorentz transformation based on the invariance of the relations between inertial frames, " *The principle of relativity is first stated in general terms, leading to the idea of equivalent frames of reference connected through inertial transformations obeying a group law. … This is the point of view from which I intend to*

criticize on the overemphasized role of the speed of light in the foundations of special relativity." He states, " *The Lorentz case is characterized by a parameter with the dimensions of a velocity which is a universal constant associated with the very structure of space-time*".

1.5 THE EST DIAGRAM IN TERMS OF THE ORIENTATION ANGLE ϕ.

The aberration $\left(\dfrac{\pi}{2} - \theta\right)$ represents a body's orientation in spacetime and we will call it as the orientation angle ϕ. Similar to studying relativistic variations as a function of θ, ($f(\theta)$), we can alternatively study it as a $f(\phi)$.
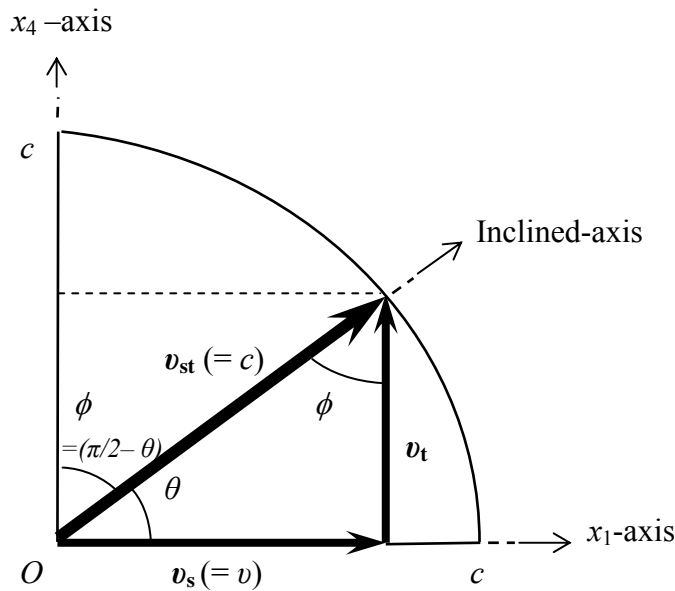

Figure 5: The EST diagram in terms of orientation angle ϕ.

From *Fig 5* , the relationship between ϕ, v_s and v_{st} is $sin\ \phi = \dfrac{|v_s|}{|v_{st}|}$

Since $|v_s| = v$ and $|v_{st}| = c$, $sin\ \phi = \dfrac{v}{c}$ .... **Eqn 12**

Applying trigonometric identities, from *Eqn 12* ,

$$sec\ \phi = \frac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}} \quad \text{and}\quad tan\ \phi = \frac{v/c}{\sqrt{1 - \dfrac{v^2}{c^2}}}$$

The Lorentz transformation equations re-expressed as a $f(\phi)$ in trigonometric form is

$x_1' = (sec \ \phi) \ x_1 - (tan \ \phi) \ x_4$ …… 1st Eqn

$x_4' = -(tan \ \phi) \ x_1 + (sec \ \phi) \ x_4$ ….. 4th Eqn

By postulating $sin \ \phi = \dfrac{v}{c}$, *Majernik* [8] has expressed the Lorentz transformation equations in trigonometric form as a $f(\phi)$ and alternatively as $f(\theta)$. He states, *" The above trigonometric form of the Lorentz transformation is interesting not only from a formal point of view, but may serve as a starting point for expressing other relativistic quantities in terms of trigonometric functions and for the formulation of relativistic relations in the language of trigonometry."* Although a physical interpretation for ϕ is not shown, by investigating [9] on photons received from a moving body, it is shown that *Majernik's* space-time angle ϕ is an aberration angle which corresponds with our model. This relationship $sin \ \phi = v/c$ also appears in Loedel's diagram[10]. By investigating on the spherical wavefront of a light pulse [11], a 4-coordinate manifold of SR has been modeled governed by the functions of a circle.

The relativistic variations expressed as a $f(\phi)$ are :-

$$time \ variation \ ratio = clockrates \ ratio \ = \ cos \ \phi = \sqrt{1 - \dfrac{v^2}{c^2}} \quad ……. \textbf{\textit{Eqn 13}}$$

The inverse is *time dilation ratio* = $sec \ \phi$

$$space \ variation \ ratio \ = \ cos \ \phi \ = \sqrt{1 - \dfrac{v^2}{c^2}} \quad …………. \textbf{\textit{Eqn 14}}$$

When $v << c$, $\phi \approx 0$ and $cos \ \phi \approx 1$ and $sec \ \phi \approx 1$. Thus the variations are negligible for this case. Applying Pythagoras theorem (*Fig 5*), the relationship between v_{st} (= c) ; $v_s = v = c \ sin \ \phi$ and $v_t = c \ cos \ \phi$ is the identity $sin^2\phi + cos^2\phi = 1$. When $v = 0$, $\phi = 0$; when $v = c$, $\phi = \pi/2$ and when $v = -c$, $\phi = -\pi/2$. The ϕ range corresponding to $-c \le v \le c$ is $-\dfrac{\pi}{2} \le \phi \le \dfrac{\pi}{2}$. In this case $v = c \ sin \ \phi$ and $cos \ \phi$ (and $sec \ \phi$) for both receding and approaching motion remains the same (+ve) consistent with the variations to be independent of these directions.

In the EST diagram, a body moves along an inclined trajectory at an angle ϕ subtended between the vertical time –axis of the observer and the inclined time-axis of the observed body. Moving bodies have been investigated [13] in terms of trajectories at an angle φ (same as ϕ) between the time-axes of an observer and observed body with expressions $V = sin \ \varphi$ and $\sqrt{(1 - V^2)} = cos \ \varphi$ (where $c = 1$) consistent with our model. In studying appearances at relativistic speeds [12] by investigating how photons emitted from a moving body are received, the observed space

variation is described in terms of an angular rotation $= sin^{-1} \dfrac{v}{c} = cos^{-1} \sqrt{1 - \dfrac{v^2}{c^2}}$ which corresponds with the orientation angle ϕ in our model. It is of interest that the work by [9], [11] and [12] studying on photons received from a moving body based on the constancy of light velocity c produces results consistent with the EST diagram.

1.6 THE EST DIAGRAM OFFERS AS A VIABLE ALTERNATIVE.

From the 2 postulates of ESR, a velocity vector spacetime model, the EST diagram, was formulated. Compared to the ST diagram as formulated from SR, (a) the 1st postulate is applied solely based on relativistic velocities thereby avoiding the background dependent [14] position coordinates and (b) the 2nd postulate uses the constancy of velocity in spacetime c for all inertial frames instead of restricting to the constancy of velocity (in space) c of light. Since ESR is based on a re-interpretation of the SR postulates applying broader relativity principles, the derived equations are consistent with SR.

With a real 4th - dimensional time parameter, the EST diagram has a Euclidean (++++) metric. The relativistic variations were expressed in terms of a real spacetime angle θ (or alternatively the orientation angle ϕ) governed by a circular function. The Lorentz transformation equations were expressed in trigonometric form using the angular velocity parameter θ or ϕ instead of in hyperbolic form using rapidity α. It is with interest we note the de Broglie waves can be expressed in terms of ϕ in trigonometric form[8] with the relativistic relationships correlated graphically by a single diagram. Also based upon a wave field system[15], a diagram corresponding with the EST diagram has been presented.

In conclusion, the consistency of the EST diagram with Lorentz transformation equations offers a new geometrical tool to investigate relativistic observations and encourages considerations on its viability to serve as a convenient alternative to the ST diagram.

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