A EUCLIDEAN SPACE ANALOGUE OF A MOVING BODY.

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Abstract

In the Euclidean Space-Time (EST) diagram, the 4th velocity represents the velocity component of a body along the time dimension. By transforming the 4th velocity as analogous to a velocity component along a space dimension, a moving body is modeled in terms of a 4-Euclidean space analogue. The advantage is applying this transformed EST diagram, called the Euclidean space (ES) diagram, relativistic dynamics can be studied in close correspondence to classical physics.

1.0 Introduction.

Based on a Euclidean interpretation of special relativity, a geometrical model of spacetime was formulated with a moving body modeled in terms of 4-Euclidean space-time (EST), called the EST diagram[1]. The EST diagram is a velocity vector based model with the 4th velocity representing the velocity component of a body along the time dimension. This suggests by transforming the 4th velocity in this diagram as analogous to a velocity component along a space dimension, it would be viable to model a moving body in terms of a 4-Euclidean space (ES) analogue diagram. The advantage of transforming the EST diagram is that dynamics can conveniently be studied in close correspondence to classical physics applying this ES diagram.

2.0 Transforming from EST to ES diagram.

In the EST diagram, the two components of the velocity in spacetime vector $\boldsymbol{v}_{\mathbf{st}}$ are the velocity in space $\boldsymbol{v}_s$ and velocity in time $\boldsymbol{v}_s$ vectors along the x_1 and x_4-axes respectively. The velocity vector addition is

$$\boldsymbol{v}_{\mathbf{st}} = \boldsymbol{v}_{\mathbf{s}} + \boldsymbol{v}_{\mathbf{t}}$$

This implies by transforming the velocity component along the x_4-axis time dimension in the EST diagram as analogous to a velocity component in a space dimension, a moving body can be modeled completely in terms of velocity in space components. With this transformation, the velocity in time vector is transformed as analogous to a velocity in space vector. The x_4-axis is an extension of the x_1-x_2-x_3-axes of usual Euclidean space and a body is studied in terms of its velocity in 4- Euclidean space analogue. In this ES diagram, the 4th velocity is represented by a transformed velocity in space vector, $\boldsymbol{V}_\mathbf{s}$. In the EST diagram a body is orientated at an angle ϕ that uniquely corresponds to its velocity υ. This implies the velocity υ along the x_1-axis in the ES diagram is the observational viewpoint of a body moving at velocity V along the x_1'-axis (inclined at an angle ϕ from the x_1-axis) in 4-Euclidean space. Since V is with reference to a Euclidean space analogue, it represents the rate of a body's proper displacement. We will call its vector as the proper velocity vector $\boldsymbol{V}$. The 2 components of $\boldsymbol{V}$ in the ES diagram are $\boldsymbol{\upsilon_s}$ and $\boldsymbol{V}_\mathbf{s}$.

Its vector addition is $\boldsymbol{V} = \boldsymbol{\upsilon_s} + \boldsymbol{V}_\mathrm{s}$ …… *Eqn 1*

For convenience we will call $\boldsymbol{\upsilon_s}$, $\boldsymbol{V}_\mathbf{s}$ and $\boldsymbol{V}$ as the conventional , transformed and proper velocity vectors with their scalar quantity magnitudes $|\boldsymbol{\upsilon_s}|$; $|\boldsymbol{V}_\mathbf{s}|$ and $|\boldsymbol{V}|$ as υ_s , V_s and V respectively.

Fig 1 shows a typical EST diagram within the circular quadrant and its corresponding ES diagram.

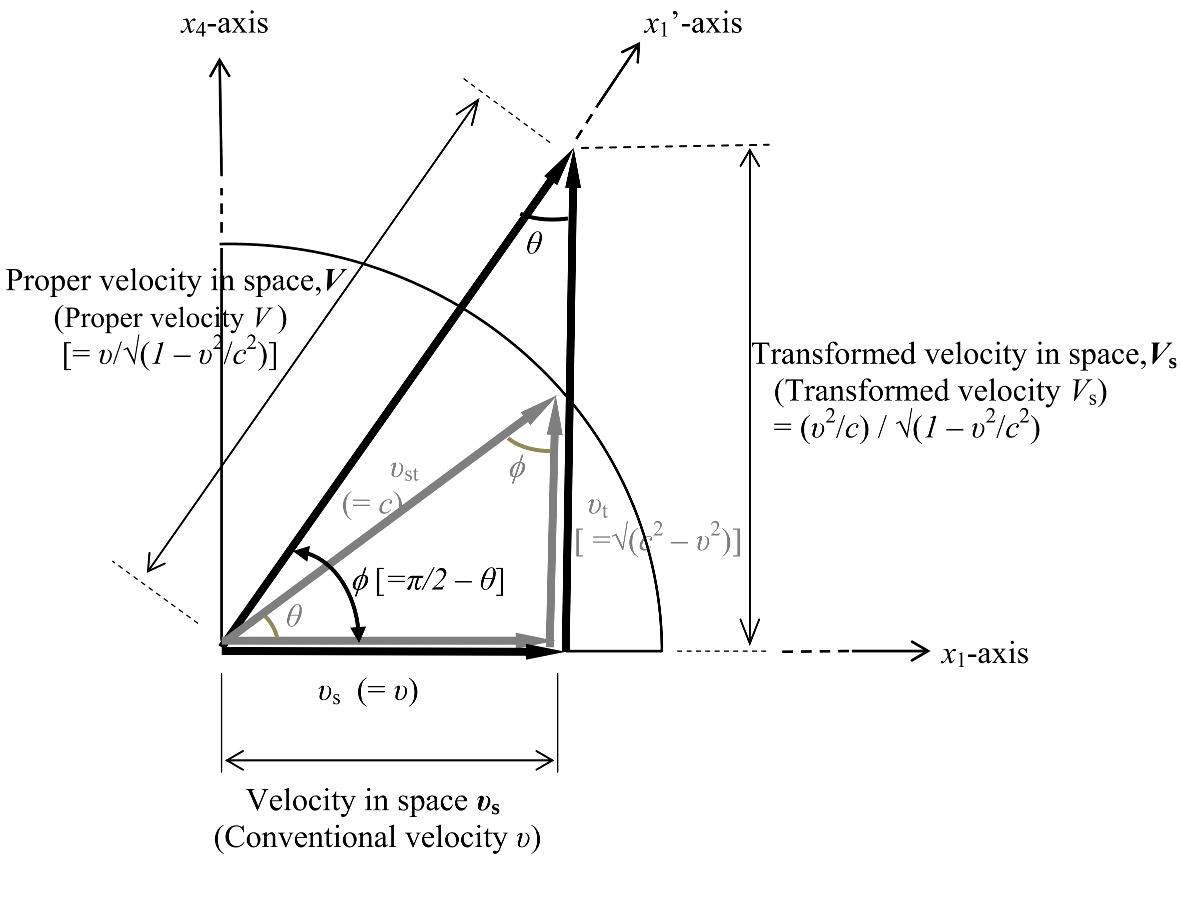

Figure 1 : The ES diagram with its corresponding EST diagram.

Both the EST and its ES diagram are similar triangles and the ratio of the magnitudes of their corresponding velocity vectors are the same.

Thus $\dfrac{|\boldsymbol{V}_s|}{|\boldsymbol{\upsilon}_s|} = \dfrac{|\boldsymbol{\upsilon}_s|}{|\boldsymbol{\upsilon}_t|}$

$$V_s = \frac{\upsilon_s^{\,2}}{\upsilon_t}$$

Substituting $\upsilon_s = \upsilon$ and $\upsilon_t = \sqrt{c^2 - \upsilon^2}$

$$V_s = \frac{\upsilon^2 / c}{\sqrt{1 - \dfrac{\upsilon^2}{c^2}}} \qquad \text{…….. } \textit{Eqn 2}$$

From *Eqn 1*, $|\boldsymbol{V}|^2 = |\boldsymbol{\upsilon}_s|^2 + |\boldsymbol{V}_s|^2$

Thus $V = \sqrt{\upsilon_s^{\,2} + V_s^{\,2}}$ …… *Eqn 3*

Substituting from *Eqn 2* into *Eqn 3*,

$$V = \frac{\upsilon}{\sqrt{1 - \dfrac{\upsilon^2}{c^2}}} \qquad \text{…… } \textit{Eqn 4}$$

It is evident that the factor that transforms the corresponding sides of the EST to the ES diagram is $\dfrac{\upsilon}{\sqrt{c^2 - \upsilon^2}}$. Compared to the EST diagram where the hypotenuse, $\upsilon_{st} = c$, a constant ; in the ES diagram, $0 < V < \infty$. As υ_s increases, υ_t in the EST diagram decreases but V_s in the ES diagram increases. As $\upsilon_s \rightarrow c$, $V_s \rightarrow \infty$. Since $\upsilon_t = \sqrt{c^2 - \upsilon^2}$ and $\upsilon_s = \upsilon$, when $\sqrt{c^2 - \upsilon^2} = \upsilon$ both triangles are identical. Solving the equation, $\upsilon = \dfrac{c}{\sqrt{2}}$ ($\approx 0.71c$). For the case $\upsilon > \dfrac{c}{\sqrt{2}}$, the EST diagram is smaller than its corresponding ES diagram as shown in *Fig 1* and reversed when $\upsilon < \dfrac{c}{\sqrt{2}}$. Since $\upsilon = c \sin \phi = c \cos \theta$, substituting into *Eqn 2* and *4*, $V_s = c \sin \phi \tan \phi = c \cos \theta \cot \theta$ and $V = c \tan \phi = c \cot \theta$. The relationships are consistent with the required sign changes for receding and approaching motion.

3.0 The Euclidean 4-vector of the ES diagram.

For convenience we fixed $\boldsymbol{\upsilon_s}$ to the x_1-axis. Expressing the vector addition for any direction in x_1-x_2-x_3-axes of space,

$$\boldsymbol{\upsilon_{s}} = \boldsymbol{\upsilon_{x1}} + \boldsymbol{\upsilon_{x2}} + \boldsymbol{\upsilon_{x3}}$$

$$|\boldsymbol{\upsilon_{s}}|^2 = |\boldsymbol{\upsilon_{x1}}|^2 + |\boldsymbol{\upsilon_{x2}}|^2 + |\boldsymbol{\upsilon_{x3}}|^2$$

$$\upsilon_{s} = \sqrt{\upsilon_{x1}^{\ 2} + \upsilon_{x2}^{\ 2} + \upsilon_{x3}^{\ 2}}$$

The vector relationship for $\boldsymbol{V_s}$ along the x_4-axis of transformed space is

$$\boldsymbol{V_{s}} = \boldsymbol{\upsilon_{x4}}$$

$$|\boldsymbol{V_{s}}| = |\boldsymbol{\upsilon_{x4}}|$$

$$V_{s} = \upsilon_{x4}$$

Expressing the velocity components in terms of the intervals for displacement along the x-axis, dx and proper time dt,

$$\upsilon_{x1} = \frac{dx_1}{dt}; \ \ \upsilon_{x2} = \frac{dx_2}{dt}; \ \ \upsilon_{x3} = \frac{dx_3}{dt}; \ \ \upsilon_{x4} = \frac{dx_4}{dt}$$

Eqn 1 expressed in terms of the Euclidean 4-vector of the ES diagram is

$$\boldsymbol{V} = \boldsymbol{\upsilon_{x1}} + \boldsymbol{\upsilon_{x2}} + \boldsymbol{\upsilon_{x3}} + \boldsymbol{\upsilon_{x4}}$$

$$|\boldsymbol{V}|^2 = |\boldsymbol{\upsilon_{x1}}|^2 + |\boldsymbol{\upsilon_{x2}}|^2 + |\boldsymbol{\upsilon_{x3}}|^2 + |\boldsymbol{\upsilon_{x4}}|^2$$

$$V^2 = \upsilon_{x1}^{\ 2} + \upsilon_{x2}^{\ 2} + \upsilon_{x3}^{\ 2} + \upsilon_{x4}^{\ 2} \quad \text{..........} \ \textit{Eqn 5}$$

Substituting for υ_{x1} ; υ_{x2} ; υ_{x3} ; υ_{x4} into *Eqn 5*

$$V^2 = \left(\frac{dx_1}{dt}\right)^2 + \left(\frac{dx_2}{dt}\right)^2 + \left(\frac{dx_3}{dt}\right)^2 + \left(\frac{dx_4}{dt}\right)^2 \quad \text{.........} \ \textit{Eqn 6}$$

The left hand side of *Eqn 6* is a variant as compared to the 4-velocity component of the EST diagram where υ_{st} is an invariant c by postulate.

4.0 Space and time variation.

In the ES diagram, the velocity υ along the x_1-axis is the observational viewpoint of the proper velocity V of a frame along the x_1'-axis in 4-Euclidean space. Resolving the frame's proper length L_o inclined at an angle ϕ from the x_1-axis, the observed length is equal to $L_o \cos \phi$.

$$\text{Thus} \quad \textit{Length contraction ratio} = \frac{L_o \cos \phi}{L_o}$$

$$= \cos \phi$$

The 'velocity in time' vector in the EST diagram has been transformed into a velocity in space analogue vector. Thus in the ES diagram, a frame is studied as moving in 4-Euclidean space analogue and the time (clockrates) variation in this case is equal to the ratio of the magnitudes of $\boldsymbol{\upsilon}_s$ and $\boldsymbol{V}$ vectors.

$$\begin{aligned} \textit{Time variation ratio} &= \frac{|\boldsymbol{\upsilon}_s|}{|\boldsymbol{V}|} \\ &= \frac{\upsilon}{V} \\ &= \frac{c \sin \phi}{c \tan \phi} \\ &= \cos \phi \end{aligned}$$

Expressed in terms of time dilation instead of clockrates ratio,

$$\textit{Time dilation ratio} = \sec \phi$$

Applying the ES diagram, a moving body is studied in terms of a Euclidean metric with mass remaining a constant corresponding to *Montanus* [2]. Also the Lorentz transformation is due to a real rotation ϕ in 4-Euclidean space which corresponds to *Gersten* [3] treating the Lorentz transformation as a 4D rotation in Euclidean space. For the case $\upsilon \ll c$, $\phi \approx 0$ and the Lorentz transformation reduces to Galilean transformation to the first approximation. When $\upsilon \to c$, $\phi \to \pi/2$ and $\cos \phi \to 0$. For this case, the length contraction and clockrates approaches 0 (or time dilation $\sec \phi \to \infty$). The velocity υ never exceeds c, implying an effect cannot precede cause consistent with causality requirements. Substituting $\phi = \pi/2 - \theta$, the length contraction and time dilation are alternatively expressed as $\sin \theta$ and $\operatorname{cosec} \theta$ respectively.

5.0 The Euclidean Space (ES) diagram.

In the ES diagram, the displacement along the inclined x_1'-axis is proper displacement, S, and its component along the x_1-axis is observed displacement, s. The rate of proper displacement, dS/dt

is equal to proper velocity V . The rate of observed displacement ds/dt is the conventional (observed) velocity υ. Due to observational limitations, measurements are restricted to observed displacement ds and not dS. Since $dS \cos \phi = ds$ and the moving and rest frame clock readings ratio, $d\tau/dt$, is equal to time variation, $\cos \phi$, therefore $V= ds/d\tau$. *Leblond* [4] has similarly shown this operational procedure to determine the proper velocity V (which he calls celerity) as the observed displacement ds per unit time interval of the moving clock, $d\tau$.

$$\textit{Conventional velocity} \;=\; \upsilon \;=\; \frac{ds}{dt} \;=\; c \sin \phi \;=\; c \cos \theta$$

$$\textit{Proper velocity} \;=\; V \;=\; \frac{dS}{dt} \;=\; \frac{ds}{d\tau} = c \tan \phi = c \cot \theta$$

From *Eqn 4*, as υ ranges from 0 to c, V correspondingly ranges from 0 to ∞. Although υ is limited by c, there is no limitation for V which is an advantage because singularities are pushed to infinity. When $\upsilon \ll c$, $V \approx \upsilon$ and also $\phi \approx 0$ and $\theta \approx \pi/2$. For this case the dynamical equations reduces to the classical form. In conclusion, since the ES diagram is a 4- Euclidean space analogue of a moving body, it offers as a convenient model to study dynamics in close correspondence to classical physics.

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4. Leblond, JM Levy ; *Speeds*, Am J Phys, 48(5), 345-347, 1980.

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