

# THE ADVANTAGE OF EUCLIDEAN INTERPRETATION OF RELATIVITY.

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## ABSTRACT

The geometrical formulation based on a euclidean interpretation of special relativity is briefly reviewed. Its consistency in satisfying many of the relativistic requirements is discussed. The advantage of the euclidean model to investigate relativistic variations conveniently as a function of a real 4-rotation in trigonometric form is shown. Its possible usage as a viable alternative to the standard interpretation is considered.

### **1.0 Euclidean special relativity.**

Based on a euclidean interpretation of special relativity , called euclidean special relativity (ESR), a circular function geometrical model of space-time, the euclidean space-time (EST) diagram was formulated [1]. The EST diagram is a 4-velocity vector model along the orthogonal space  $x_1$ - $x_2$ - $x_3$ -axes and time  $x_4$ -axis. The  $x_4$ -axis is real and not imaginary as is the case with the space-time (ST) diagram formulated from special relativity (SR). Thus the metric in the EST diagram is (++++) euclidean compared with the non-euclidean (++++-) metric in the ST diagram. As a result, the Lorentz transformation equations are expressed in terms of a real 4-rotation as a function of the space-time angle  $\theta$  or alternatively the orientation angle  $\phi$  in simple trigonometric form instead of a complex 4-rotation expressed in hyperbolic form as a function of rapidity  $\alpha$ .

Subsequently, the velocity-in-time vector component of a frame along the  $x_4$ -axis was transformed analogous to a velocity-in-space vector component. With this the forth  $x_4$ -axis in the EST diagram was transformed as an extension of the classically assumed  $x_1$ - $x_2$ - $x_3$ -axes of euclidean space. In the resultant diagram, called the Euclidean space (ES) diagram, a body is studied as moving at proper velocity  $V$  in 4-euclidean space in close correspondence to classical physics.

### **2.0 Consistency with special relativity.**

The relativistic variation in space,  $s$  and time,  $t$  , derived from the euclidean model were consistent with the standard equations. The momentum  $p$  and energy  $E$  equation were derived without invoking the problematic relativistic mass concept. Also the consistency of the

momentum-energy  $p$ - $E$  relationship, the mass-energy  $m$ - $E$  equivalence and the frequency  $\nu$  variation were shown. That a body approaches a singularity as velocity  $v$  approaches  $c$  and for the case  $v \ll c$ , the equations reduces to classical physics was shown. Since velocity  $v$  of a body never exceeds light velocity  $c$ , this is consistent with causality requirements.

From the euclidean model, the velocity addition formula for collinear frames was deduced as the sum of proper velocities  $V$ . Although this deduced formula is consistent with satisfying the relativistic requirements, it slightly deviates from the standard one given as the sum of rapidities  $\alpha$ . The interpretation problems in conclusively verifying the addition formula was discussed. It is of interest the work done purely as a mathematical exercise by *Majernik*[2] expressing the relativistic variations as a function of an angular velocity parameter  $\phi$  and *MathPages*[3] in offering an alternative composition speed formula exactly corresponds with euclidean relativity.

### 3.0 The advantages of the euclidean model.

An advantage of the euclidean metric model is that the relativistic variations are conveniently expressed as a function of a real 4-rotation which provides an easier co-relation of the variations with geometry. For example the length contraction is explained as due to a real angular rotation from an observer's line of sight without confusion whether it is Fitzgerald contracted. Thus the physical significance of the angular velocity parameter  $\phi$  (or  $\theta$ ) is more defined compared to rapidity  $\alpha$  in the standard expressions.

In this euclidean model, relativistic dynamics is conveniently studied in close correspondence with classical physics in terms of the unbounded proper velocity  $V$  range. Consequently the description of singularities are pushed to an infinite limit instead of the finite  $c$  restriction using conventional velocity  $v$ . With both mass  $m$  and  $c$  as constants, a body's momentum varies in direct proportion with proper velocity  $V$  corresponding to the linear relationship between momentum and velocity  $v$  in classical physics. With momentum  $p$  and energy  $E$  of a body expressed as  $p = mc \tan \phi = mc \cot \theta$  and  $E = mc^2 \sec \phi = mc^2 \operatorname{cosec} \theta$  respectively, the variation is geometrically explained as due to the real 4-rotation without invoking the problematic relativistic mass concept. Therefore, expressing the momentum  $p=Mv$  and energy  $E = Mc^2$  where  $M$  is the relativistic mass of a moving body with mass  $m$  (at rest) is avoided, consistent with Einstein's view [4].

### 4.0 Summary of the relativistic expressions.

In this euclidean model, a moving body is conveniently studied as a function of a real 4-rotation  $\phi$  (or  $\theta$ ) in trigonometric form with the relativistic expressions as:-

- (a) Conventional velocity  $v = c \sin \phi = c \cos \theta$  ;
- (b) Proper velocity  $V = c \tan \phi = c \cot \theta$  ;
- (c) Transformed velocity in space  $V_s = c \sin \phi \tan \phi = c \cos \theta \cot \theta$  ;
- (d) Time variation in terms of clock rates ratio =  $\cos \phi = \sin \theta$ .  
(If in terms of time dilation,  $TD$ , the ratio is reversed ,  $TD = \sec \phi = \operatorname{cosec} \theta$  ) ;

- (e) Space variation,  $\frac{L}{L_0} = \cos \phi = \sin \theta$  ;
- (f) Momentum variation,  $p = mc \tan \phi = mc \cot \theta$  ;
- (g) Energy variation,  $E = mc^2 \sec \phi = mc^2 \operatorname{cosec} \theta$  ;
- (h) Frequency variation ,  $\frac{V_0}{\nu} = (\sec \phi + \tan \phi) = (\operatorname{cosec} \theta + \cot \theta)$  ;
- (i) Velocity addition formula,  $\tan \phi_3 = (\tan \phi_1 + \tan \phi_2)$  ;  $\cot \theta_3 = (\cot \theta_1 + \cot \theta_2)$

The relationship between velocity in time  $v_t = c \cos \phi = c \sin \theta$  and velocity in space  $v_s = c \sin \phi = c \cos \theta$  is given by the trigonometric identity  $\cos^2 \phi + \sin^2 \phi = 1$  and  $\sin^2 \theta + \cos^2 \theta = 1$ . Between momentum  $p = mc \tan \phi = mc \cot \theta$  and energy  $E = mc^2 \sec \phi = mc^2 \operatorname{cosec} \theta$ , it is given by the trigonometric identity  $\sec^2 \phi - \tan^2 \phi = 1$  and  $\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$ .

*Majernik* has stated that , “ *Relativistic considerations have played an important role in the introduction of the relation  $\lambda = h/p$  between the wavelength  $\lambda$  of the de Broglie waves and the momentum  $p$  of a particle. Although de Broglie has introduced his phase waves as an intrinsically relativistic concept, linked intimately with the Lorentz transformation, this fact is often not sufficiently emphasised.*” The correlation between relativistic quantities for particles and the corresponding de Broglie waves is shown in terms of  $\phi$  in trigonometric form. This suggests the applicability of the euclidean model in studying this waves as a function of the real 4-rotation  $\phi$ .

In conclusion the geometrical model as formulated based on a euclidean interpretation of relativity encourages considerations on its viability to serve as a convenient alternative from the standard interpretation.

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