RELATIVISTIC FREQUENCY IN TRIGONOMETRIC TERMS.

S. Kanagaraj
Euclidean Relativity
s.kana.raj@gmail.com
(24 October 2009)

Abstract

The frequency variation equation is derived in relation to 4-Euclidean metric diagram. The relativistic equations are expressed in terms of a real angular rotation ϕ (or alternatively θ) in trigonometric form. The very small relativistic effect for sound waves due to time dilation is also shown.

1.0 Introduction.

Based on a Euclidean interpretation of special relativity (SR), the Euclidean space-time (EST) diagram was formulated[1] and subsequently transformed into a Euclidean space (ES) analogue of a moving body[2]. The relativistic space, time, momentum and energy variation equations were derived expressed in trigonometric form as a function of the spacetime angle θ (or alternatively the orientation ϕ). We next express the relativistic frequency as a function of θ (or ϕ) in trigonometric form.

2.0 Effects contributing to frequency variation.

When a wavesource with rest frequency ν_o is receding at velocity υ, the effect of its position changes causes the wavelength λ received to be larger than when at rest λ_o . The time interval between wavefronts, period T, is larger than T_o when at rest. For this case, $\lambda > \lambda_o$; $T > T_o$ and since $\nu = \dfrac{1}{T}$, $\nu < \nu_o$. For approaching case $\lambda < \lambda_o$; $T < T_o$ and $\nu > \nu_o$. The wavelength change, $\Delta\lambda$, with λ_o ratio due to this effect is[1] $\dfrac{\Delta\lambda}{\lambda_o} = \dfrac{\upsilon}{c_p}$ with c_p , the wave propagation velocity. Re-arranging, $\dfrac{\lambda}{\lambda_o} = 1 \pm \dfrac{\upsilon}{c_p}$, with (+) and (–) signs for receding and approaching motion respectively. Noting $\nu\lambda = c_p$ and assuming constant propagation velocity, $\dfrac{\lambda}{\lambda_o} = \dfrac{\nu_o}{\nu} = \dfrac{T}{T_o}$.

From EST diagram, $\cos\theta = \dfrac{\upsilon}{c}$, where c is the invariant velocity in spacetime. If light propagation velocity is c_{light} then $c_p = c_{light} = c$ and

[1] This relationship for a wave source moving along a straight path from a stationary observer is derived with ease from first principles.

$$\frac{\nu_o}{\nu} = 1 + \cos\,\theta \quad \ldots\ldots\ Eqn\ 1$$

When receding, υ ranges from 0 to c and the corresponding θ range is $\pi/2 \geq \theta \geq 0$. When approaching, υ ranges from 0 to $-\,c$ and θ range is $\pi/2 \leq \theta \leq \pi$. The $\cos\,\theta$ sign change is consistent with the velocity sign change for different directions.

Also due to time dilation (*TD*), a moving wavesource frequency ν is less than when at rest ν_o . The change in the periods ratio $\dfrac{T}{T_o}\left(=\dfrac{\nu_o}{\nu}\right)$ due to this effect is the *TD* ratio, thus

$$\frac{\nu_o}{\nu} = \frac{T}{T_o} = TD = \frac{1}{\sqrt{1-\dfrac{\upsilon^2}{c^2}}}\ .$$

Substituting $\cos\theta = \dfrac{\upsilon}{c}$, $\quad \dfrac{\nu_o}{\nu} = cosec\,\theta \ \ldots\ldots\ldots\ Eqn\ 2$

For receding and approaching motion, θ ranges from $\pi/2 \geq \theta \geq 0$ and $\pi/2 \leq \theta \leq \pi$ respectively with $cosec\,\theta$ sign consistent with time variation as independent of these directions.

3.0 Deriving the frequency variation equation.

The resultant frequency variation is the product of the two effects in *Eqn 1* and *2*.
For the case of a moving lightwave source,

$$\begin{aligned}\frac{\nu_o}{\nu} &= (1 + \cos\,\theta)\ cosec\,\theta \\ &= cosec\,\theta + \cot\,\theta \quad \ldots\ldots\ Eqn\ 3\end{aligned}$$

From *Eqn 3*,

$$\frac{\nu_o}{\nu} = \frac{1+\cos\theta}{\sin\theta} = \sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$$

Since $\cos\,\theta = \dfrac{\upsilon}{c}$, $\quad \dfrac{\nu_o}{\nu} = \sqrt{\dfrac{1+\dfrac{\upsilon}{c}}{1-\dfrac{\upsilon}{c}}}$ for receding motion with the signs reversed for approaching motion, consistent with the relativistic Doppler frequency shift for light. For a soundwave source,

$$\frac{\nu_o}{\nu} = \left(1 \pm \frac{v}{c_{sound}}\right) cosec\ \theta \qquad \ldots\ldots\ Eqn\ 4$$

where c_{sound} is the soundwave propagation velocity. As c is the invariant velocity in spacetime , the effect due to time dilation remains as $cosec\ \theta$. *Eqn 4* which accounts for time dilation is consistent with the sound Doppler effect [3]. Since $cosec\,\theta = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = TD$ and for this case $v \lll c$, $cosec\ \theta$ is very close to 1 . The time dilation correction factor is negligible [4] and reduces to classical Doppler shift equation.

Since $cos\ \theta = \frac{v}{c} = sin\ \phi$, substituting into *Eqn 3*, the equation is expressed alternatively as a function of the orientation angle ϕ with the sign change again consistent with relativistic requirements,

$$\frac{\nu_o}{\nu} = sec\ \phi + tan\ \phi \quad \ldots\ldots\ldots\ldots\ldots\ Eqn\ 5$$

In conclusion, the derived expressions provides an alternative approach to investigate frequency variations as a function of θ or ϕ in trigonometric terms and offers new avenues in studying relativistic dynamics.

<u>REFERENCES</u>

1. Kanagaraj, S : *Euclidean alternative to Minkowski spacetime diagram.*, http://www.euclideanrelativity.net , 12 Aug 2009.

2. Kanagaraj, S : *Euclidean space analogue of a moving body.*, http://www.euclideanrelativity.net, 2 Sept 2009

3. Reynolds, Robert : *Doppler effect for sound via classical and relativistic space-time diagrams*, Am.J.Phys.,58(4),390-394,1990.

4. Bachman, R A : *Relativistic acoustic Doppler effect*, Am.J.Phys.,50,816-818,1982.

---------------------- *END* ----------------------